Math 113 (Calculus II)
Test 1 Form A KEY

Multiple Choice. Fill in the answer to each problem on your scantron. Make sure your name, section and instructor is on your scantron.

1. Find $\displaystyle\int_4^9 \frac{\ln y}{\sqrt{y}}\,dy$

   a) $6\ln 9 - 2\ln 4 + 4$ b) $\frac{3}{2}\ln 9 - \ln 4 + 2$ c) $6\ln 9 - 4\ln 4 - 2$

   d) $6\ln 9 - 4\ln 4 - 4$ e) $\frac{3}{2}\ln 9 - \ln 4 - 2$ f) $6\ln 9 - 2\ln 4 - 4$

   g) None of the above

   Solution: d)

2. Rotate the area between the curves

$$x = 0, \quad y = x^{1/3}, \quad y = 1$$

   about $y = -1$. The volume is

   a) $\frac{1}{10}\pi$ b) $\frac{3}{10}\pi$ c) $\frac{6}{10}\pi$ d) $\frac{9}{10}\pi$

   e) π f) 3π g) None of the above.

   Solution: d)

3. Rotate the area between $y = x$ and $y = x^2$ about $x = a$ $(a > 1)$. For what value of a is the volume $\frac{\pi}{2}$?

   a) 1 b) 2 c) 3 d) 4

   e) 5 f) 6 g) None of the above.

   Solution: b)

4. Evaluate $\displaystyle\int_0^4 3\sqrt{16 - x^2}\,dx$ by interpreting it as an area.

a) π b) 2π c) 3π d) 4π

e) 12π f) 24π g) None of the above.

Solution: e)

5. A force of 12 lb is required to hold a spring stretched 2 ft beyond its natural length. How much work is done in stretching it from its natural length to 4 ft beyond its natural length?

a) 12 lb b) 24 lb c) 36 lb

d) 48 lb e) 48 ft-lb f) 36 ft-lb

g) 24 ft-lb h) 12 ft-lb

Solution: e)

6. Find the average value of the function $f(\theta) = \sin(\theta)\cos(\theta)$ over the interval $[0, \frac{\pi}{3}]$.

a) $\frac{3}{8}$ b) $\frac{3}{4}$ c) $\frac{3}{2}$

d) $\frac{9}{8\pi}$ e) $\frac{9}{4\pi}$ f) $\frac{\sqrt{3}}{4}$

g) $\frac{3\sqrt{3}}{4\pi}$ h) None of the above.

Solution: d)

7. $\int_0^{\pi/2} \cos^4(2x)\ dx$

a) $\frac{3\pi}{8}$ b) $\frac{3\pi}{16}$ c) $\frac{3\pi}{32}$

d) $\frac{3\pi}{32} + \frac{9}{64}$ e) $\frac{3\pi}{16} + \frac{9}{16}$ f) $\frac{3\pi}{8} + \frac{9}{16}$

g) $\frac{3\pi}{8} + \frac{9}{8}$ h) None of the above.

Solution: b)

8. $\int_0^{\frac{\pi}{12}} \tan^3(3x)\sec^4(3x)\ dx$

a) $\frac{3\pi}{12}$ b) $\frac{3\pi}{18}$ c) $\frac{3\pi}{36}$

d) $\frac{5}{12}$ e) $\frac{5}{18}$ f) $\frac{5}{36}$

g) $\frac{2}{3}$ h) None of the above.

Solution: f)

Free response: Give your answer in the space provided. Answers not placed in this space will be ignored. 6 points each

9. (5 points) Set up the integral that represents the volume of the solid created by rotating the area between

$$y = \frac{1}{4}x^2 + 1, \quad y = x, \quad x = 0$$

about $x = 2$. Do not evaluate the integral.

Solution:

$$\int_0^2 2\pi(x-2)(\frac{1}{4}x^2 - x + 1)\,dx = 2\pi\int_0^2 1/4\,x^3 - 3/2\,x^2 + 3\,x - 2\,dx$$

10. (5 points) Set up the integral that represents the volume of the solid created by rotating the area between

$$x = y^2 + 2, \quad x = \sqrt{y}, \quad y = 0, \quad y = 2$$

about $x = -1$. Do not evaluate the integral.

Solution:

$$\int_0^2 \pi\left[(y^2+2+1)^2 - (\sqrt{y}+1)^2\right]\,dy = \pi\int_0^2 y^4 + 6\,y^2 + 8 - y - 2\sqrt{y}\,dy$$

11. (5 points) Find the area of the region bounded by the curves $x = y^2$ and $x = 8 - y^2$.

Solution: If we set $y^2 = 8 - y^2$, we have $y^2 = 4$, or $y = \pm 2$. The area is

$$\int_{-2}^2 (8 - y^2 - y^2)\,dy = \int_{-2}^2 (8 - 2y^2)\,dy = (8y - \frac{2}{3}y^3)_{-2}^2$$

$$= 16 - \frac{16}{3} - (-16 + \frac{16}{3}) = 32 - \frac{32}{3} = \frac{64}{3}.$$

12. (5 points) Find $\int x^3\sin(x^2)\ dx$.

Solution: Let $z = x^2$. then, $dz = 2x\,dx$, and

$$\int x^3\sin(x^2)\ dx = \int z\sin(z)\,\frac{dz}{2}.$$

We now use integration by parts: $u = z$, $du = dz$, $dv = \sin z\, dz$, and $v = -\cos z$. The above integral then becomes

$$\frac{1}{2}(-z\cos z + \int \cos z\, dz) = \frac{1}{2}(-z\cos z + \sin z) + C$$
$$= \frac{1}{2}(-x^2\cos x^2 + \sin x^2) + C.$$

13. (5 points) Find $\int_0^1 3e^{-s}s^2\ ds$

Solution: We need to use integration by parts twice:

Let $u = s^2$, $du = 2s\,ds$, $dv = e^{-s}\,ds$, $v = -e^{-s}$. Then the above integral is

$$3(-s^2e^{-s} + 2\int se^{-s}).$$

Using integration by parts again, we have $u = s$, $du = ds$, $dv = e^{-s}$, $v = -e^{-s}$. Thus, the above becomes

$$3(-s^2e^{-s} + 2(-se^{-s} + \int e^{-s})) + C$$
$$= -3s^2e^{-s} - 6se^{-s} - 6e^{-s} + C.$$

14. (5 points) A 24 ft chain weighs 12 lb and hangs from a ceiling. Find the work done in lifting the lower end of the chain to the ceiling so that it's level with the upper end.

Solution: After lifting the end of the chain x feet, there is $24 - x$ feet above, so the chain below is a total of x feet. However, the chain below, is doubled, so there is $x/2$ feet being held. Since the chain weighs $1/2$ lb per foot, the force acting on the lifted portion of the chain at that point is $F(x) = \frac{x}{4}$.

Since the end of the chain is lifted a distance of 24 feet, the work done is

$$\int_0^{24} \frac{x}{4}\,dx = \frac{x^2}{8}\Big|_0^{24} = \frac{24^2}{8} = \frac{9\cdot 8^2}{8} = 72 \text{ ft-lbs}.$$

15. (5 points) Given the function $f(x) = x^3$ over the interval $[0, 2]$, find c such that the average value of f is equal to $f(c)$.

Solution: The average value is

$$\frac{1}{2-0}\int_0^2 x^3\,dx = \frac{1}{2}\frac{x^4}{4}\Big|_0^2 = \frac{16}{8} = 2.$$

For $f(c) = 2$, we need $c^3 = 2$, or $c = \sqrt[3]{2}$.

16. (5 points) Find $\int \sin(3x)\cos(4x)\ dx$.

Solution:

$$\int \sin(3x)\cos(4x)\ dx = \int \frac{1}{2}(\sin(7x) - sin(x))\,dx = -\frac{\cos(7x)}{14} + \frac{\cos(x)}{2} + C$$

17. (5 points) Find the volume of the solid S that has a triangular base with vertices in the x-y plane $(0,0)$, $(1,0)$, and $(0,1)$. Cross-sections perpendicular to the y-axis are equilateral triangles.

Solution: At height y, the length across the triangle is $1-y$. The area of an equilateral triangle of side s is $\frac{\sqrt{3}}{4}s^2$. Thus, the cross sectional area is given by

$$\frac{\sqrt{3}}{4}(1-y)^2,$$

and the volume is

$$\int_0^1 \frac{\sqrt{3}}{4}(1-y)^2\,dy = -\frac{\sqrt{3}}{4}\frac{(1-y)^3}{3}\Big|_0^1$$
$$= -\frac{\sqrt{3}}{12}(0-1) = \frac{\sqrt{3}}{12}.$$

END OF EXAM